

Electrical Technology: Starting, speed control & braking of 3-phase induction motor

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1 Objectives

For any motor, it is important to know various methods of starting, speed control and braking. In this unit, popular methods of starting, speed control and braking for 3-phase induction motor are discussed.

After going through this unit, students will be able to:

1. understand the problems of starting large induction motor with full rated voltage.
2. enumerate and explain various methods of starting for cage induction motor.
3. suggest suitable method of starting a wound rotor induction motor.
4. explain various methods of speed control of induction motor.
5. compare various methods so far as starting current drawn from the supply and the starting torque are concerned.
6. understand the necessity of electrical braking.
7. explain various methods of braking of induction motor.

2 Starting of Cage induction motor

Comparison of various starting methods as enumerated below, will be carried out with respect to the full voltage starting for cage induction motor.

1. Direct On Line (DOL) starting with full voltage.
2. Reactor starting.
3. Autotransformer starting.
4. Star-Delta starting.

During the analysis, we shall

1. neglect the no load current, since it is much smaller compared to the starting current.
2. assume that the stator of the machine is delta connected.
3. assume the per phase reflected impedance at stand still condition to be $z = \sqrt{(r_1 + r'_2)^2 + (x_1 + x'_2)^2}$

2.1 Direct ON Line (DOL) starting

In this method full rated voltage is applied to start the motor as shown in figure 1.

$$\text{Let the per phase equivalent impedance be } z = \sqrt{(r_1 + r'_2)^2 + (x_1 + x'_2)^2}$$

$$\text{Let the line to line rated voltage} = V$$

$$\text{Per phase motor current, } I_{ph} = V/z$$

$$\text{Current drawn from the supply, } I_{sL}(\text{dol}) = \sqrt{3}V/z$$

$$\text{Starting torque in syn watt } T_{st}(\text{dol}) = 3I_{ph}^2 r'_2$$

$$\text{Starting torque in syn watt } T_{st}(\text{dol}) = 3V^2 r'_2 / z^2 \quad (1)$$

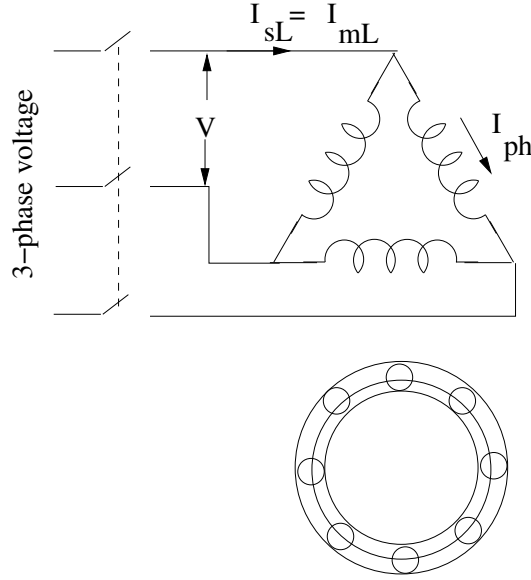


Figure 1: DOL starting

2.2 Reactor starting

In this method three equal values of reactors are connected in series with the lines to start the motor as shown in figure 2. As the motor picks up speed the external reactors are reduced to zero so that finally the motor operates at rated voltage. This method in fact employs reduced voltage starting. Let the reactor value is so chosen that the terminal voltage to the motor is xV , where $x < 1$.

$$\begin{aligned}
 \text{Let the line to line rated voltage} &= V \\
 \text{Motor per phase voltage} &= xV \\
 \text{Per phase motor current, } I_{ph} &= xV/z \\
 \text{Current drawn from the supply, } I_{sL}(\text{reactor}) &= \sqrt{3}xV/z \\
 \text{Starting torque in syn watt } T_{st}(\text{reactor}) &= 3I_{ph}^2 r'_2 \\
 \text{Starting torque in syn watt } T_{st}(\text{reactor}) &= 3x^2 V^2 r'_2 / z^2
 \end{aligned} \tag{2}$$

Comparing with the DOL starting, we note that

$$\frac{\text{current drawn from supply in reactor starting}}{\text{current drawn from supply in DOL}} = \frac{I_{sL}(\text{reactor})}{I_{sL}(\text{dol})} = x \tag{3}$$

and

$$\frac{\text{Starting torque in reactor starting}}{\text{Starting torque in DOL}} = \frac{T_{st}(\text{reactor})}{T_{st}(\text{dol})} = x^2 \tag{4}$$

Thus we see that if we adopt reactor starting, starting current drawn from the supply reduces by a factor x while the starting torque reduces by a factor of x^2 .

2.3 Autotransformer starting

In this method the motor is supplied with reduced voltage using an autotransformer as shown in figure 3. As the motor picks up speed autotransformer is by passed so that finally the motor operates at rated voltage. Let the autotransformer is so set that the terminal voltage to the motor

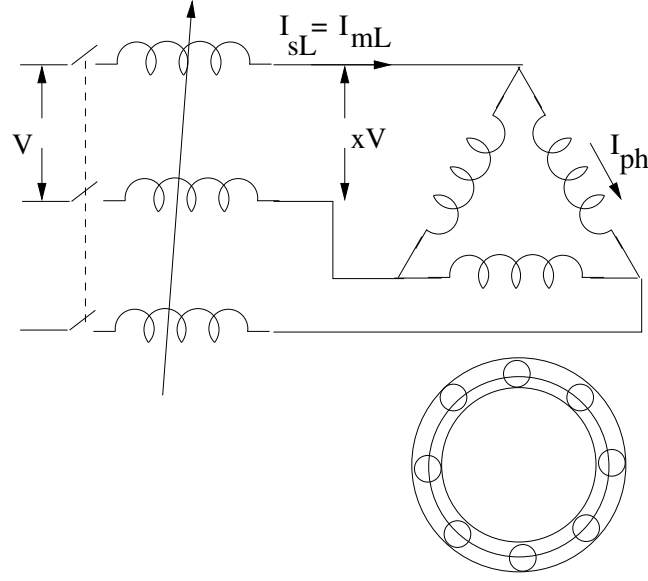


Figure 2: Starting with reactors

is xV , where $x < 1$.

$$\begin{aligned}
 \text{Let the line to line rated voltage} &= V \\
 \text{Motor per phase voltage} &= xV \\
 \text{Per phase motor current, } I_{ph} &= xV/z \\
 \text{Autotransformer secondary current, } I &= \sqrt{3}xV/z \\
 \text{Current drawn from the supply, } I_{sL}(\text{auto}) &= \sqrt{3}x^2V/z \\
 \text{Starting torque in syn watt } T_{st}(\text{auto}) &= 3I_{ph}^2 r'_2 \\
 \text{Starting torque in syn watt } T_{st}(\text{auto}) &= 3x^2V^2 r'_2 / z^2
 \end{aligned} \tag{5}$$

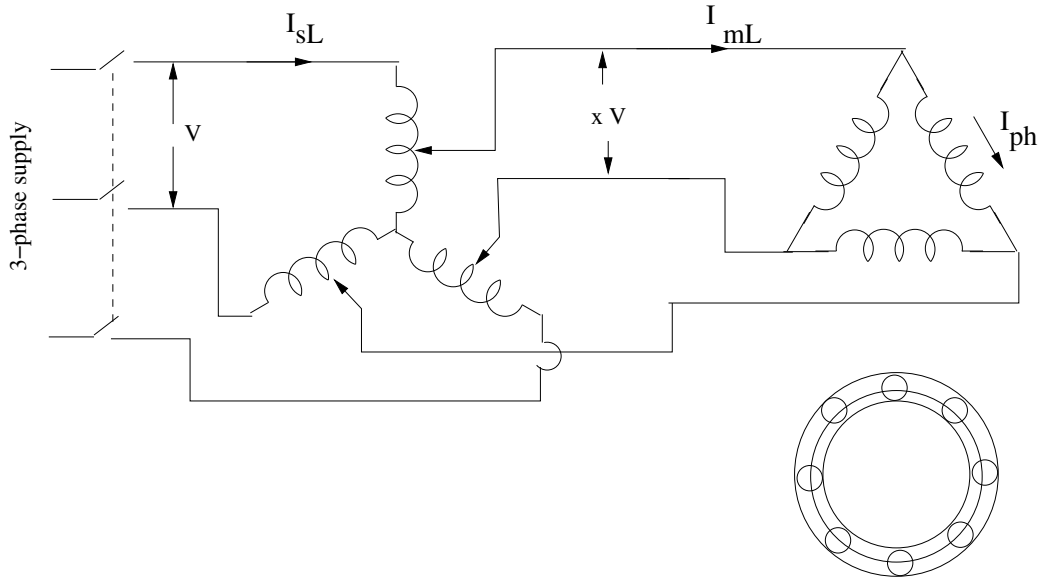


Figure 3: Starting with autotransformer

Comparing with the DOL starting, we note that

$$\frac{\text{current drawn from supply in autotransformer starting}}{\text{current drawn from supply in DOL}} = \frac{I_{sL}(\text{auto})}{I_{sL}(\text{dol})} = x^2 \tag{6}$$

and

$$\frac{\text{Starting torque in autotransformer starting}}{\text{Starting torque in DOL}} = \frac{T_{st}(\text{auto})}{T_{st}(\text{dol})} = x^2 \quad (7)$$

Thus we see that if we adopt autotransformer starting, both the starting current drawn from the supply and the starting torque reduces by a factor of x^2 .

2.4 Star-Delta starting

In this method, stator winding is connected in star at the time of starting and then switched over to delta connection after the motor attains sufficient speed. Obviously this method is applicable only for those motors whose stator coils are designed for delta connection. To implement this method all the six terminals of stator phases must be available so that desired connections star or delta can be made. Using a triple pole double throw switch, one can implement star-delta starting as shown in figure 4. The TPDT switch when connected to star position, it joins A2, B2 and C2 together giving star connection. The switch is thrown to right (delta position) after the motor picks up speed. It is also a reduced voltage starting method as voltage applied per phase at starting is $V/\sqrt{3}$ instead of V the rated phase voltage.

$$\begin{aligned} \text{Let the line to line rated voltage} &= V \\ \text{Motor per phase voltage} &= V/\sqrt{3} \\ \text{Per phase motor current, } I_{ph} &= V/\sqrt{3}z \\ \text{Current drawn from the supply, } I_{sL}(\text{stardelta}) &= V/z \\ \text{Starting torque in syn watt } T_{st}(\text{stardelta}) &= 3I_{ph}^2 r'_2 \\ &= 3(V/\sqrt{3}z)^2 r'_2 = (V^2/z^2)r'_2 \end{aligned} \quad (8)$$

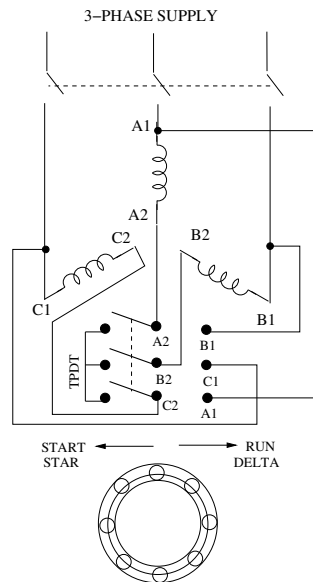


Figure 4: Starting with star-delta starter

Comparing with the DOL starting, we note that

$$\frac{\text{current drawn from supply in star-delta starting}}{\text{current drawn from supply in DOL}} = \frac{I_{sL}(\text{stardelta})}{I_{sL}(\text{dol})} = 1/3 = (1/\sqrt{3})^2 \quad (9)$$

and

$$\frac{\text{Starting torque in star-delta starting}}{\text{Starting torque in DOL}} = \frac{T_{st}(\text{stardelta})}{T_{st}(\text{dol})} = (1/\sqrt{3})^2 \quad (10)$$

Thus we see that if we adopt star-delta starting, both the starting current drawn from the supply and the starting torque reduces by a factor of $(1/\sqrt{3})^2 = 1/3$. Thus this method of starting is equivalent to an auto transformer starting with $x = 1/\sqrt{3}$

Starting torque is often expressed in terms of full load torque as follows.

$$\begin{aligned} \text{Let, phase current at starting} &= I_{st} \\ \text{phase current at full load} &= I_{fl} \\ \text{full load slip} &= s_{fl} \\ \text{full load torque in syn watt } T_{fl} &= 3I_{fl}^2(r'_2/s_{fl}) \\ \text{starting torque in syn watt } T_{st} &= 3I_{st}^2r'_2 \\ \therefore \frac{T_{st}}{T_{fl}} &= \left(\frac{I_{st}}{I_{fl}}\right)^2 s_{fl} \end{aligned} \quad (11)$$

3 Starting of slip ring motor

In case of wound rotor or slip ring induction motor, external resistance in each phase of the rotor circuit is connected. Therefore at starting, impedance offered to the supply at starting would be:

$$z = \sqrt{(r_1 + r'_2 + r'_{ext})^2 + (x_1 + x'_2)^2}$$

Thus starting current can be reduced to desired level by choosing appropriate r_{ext} . As we know, addition of external rotor resistance also improves starting torque, therefore a judicious choice of r_{ext} will not only reduce starting current but also improves starting torque. The resistance is kept at its maximum value at the time of starting and is gradually brought down to zero as the motor picks up speed.

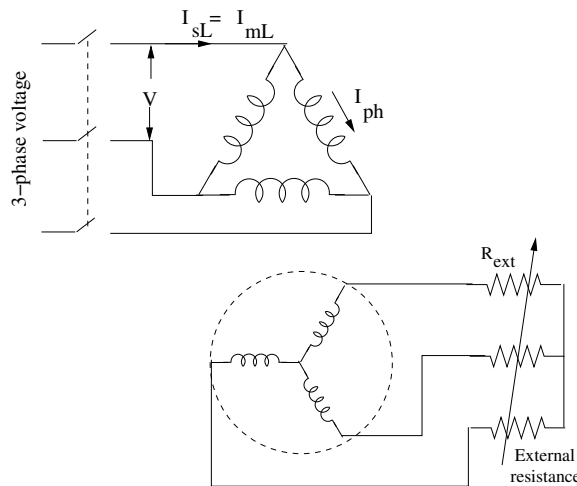


Figure 5: Starting for wound rotor motor

4 Speed control

We know that for a induction motor, variation of speed from no-load to full load is little ie., inherent speed regulation is small; that is why induction motor becomes a practically constant speed motor. However, it is often necessary to run the motor at various speeds when driving a particular mechanical load. The knowledge of the characteristic (T_L vs n) of the load plays important role in deciding which method of speed control should be adopted. In earlier days (pre power electronics era), methods adopted to control the speed of induction motor were limited. In this unit we shall briefly discuss the following methods.

1. speed control by connecting two induction motor in cascade.
2. speed control by varying the number of poles of the stator.
3. speed control by varying the supply voltage. This method is applicable to both cage and slip-ring machines.
4. speed control by varying rotor resistance. Obviously this method is applicable to slip-ring machines only.

4.1 speed control by cascading two induction motor

This is an old method- not used now a days. You may skip this method for now. To implement this two mechanically induction motors having different poles are used. One of them must be of slip-ring type. The idea is explained in Figure 6. Machine-1 is supplied from the mains at frequency f . The stator of Machine-2 is supplied from the rotor terminals of the first machine. Let us also assume that the sequence of supply is such that the rotating field produced by the stator windings of both the machines rotate in the clockwise direction. Speed of the set is assumed to be n_r and the same direction of the stator field. Now:

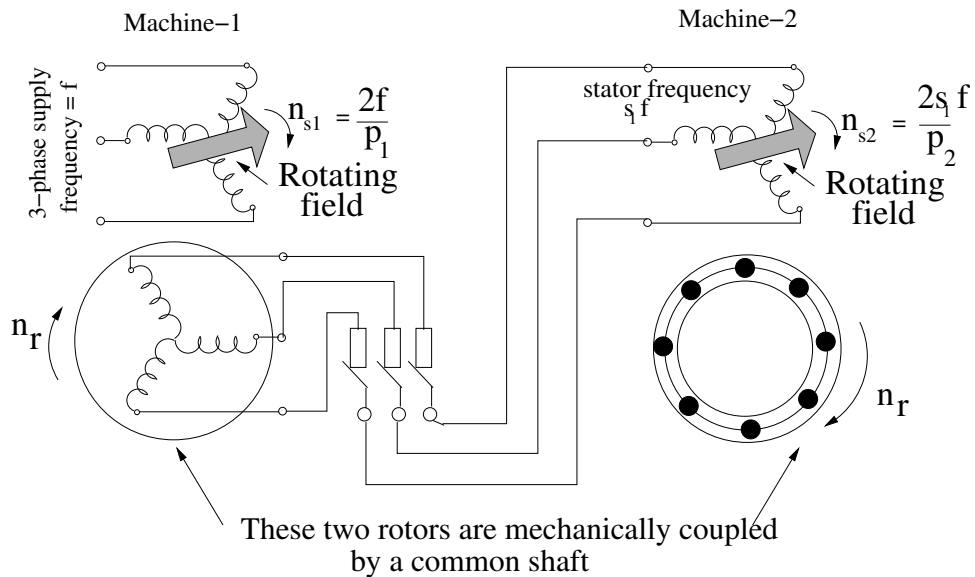


Figure 6: Speed control by controlling supply voltage

$$\text{No. of poles of machine-1} = p_1$$

$$\text{No. of poles of machine-2} = p_2$$

$$\text{Let, supply frequency of machine-1} = f$$

$$\text{Syn. speed of machine-1 } n_{s1} = 2f/p_1 \text{ in rps}$$

$$\text{Let the rotor speed of the set} = n_r$$

$$\text{Let the slip of machine-1} = s_1$$

$$\therefore n_r = (1 - s_1)n_{s1} = (1 - s_1)\frac{2f}{p_1}$$

$$\text{Now syn. speed of machine-2 } n_{s2} = 2s_1f/p_2$$

$$\therefore n_r = (1 - s_2)n_{s2} = (1 - s_2)\frac{2s_1f}{p_2}$$

Now equating n_r obtained from machine-1 and machine-2

$$(1 - s_1)\frac{2f}{p_1} = (1 - s_2)\frac{2s_1f}{p_2} \quad (12)$$

Simplifying the above equation and neglecting the product s_1s_2 we get:

$$\begin{aligned} s_1 &= \frac{p_2}{p_1 + p_2} \\ \therefore n_r &= (1 - s_1)\frac{2f}{p_1} = \frac{2f}{p_1 + p_2} \end{aligned} \quad (13)$$

There is another possibility of running the set at a speed of $n_r = \frac{2f}{p_1 - p_2}$, if the phase sequence of the supply to the stator of the second machine is reversed. In this case, stator rotating field of machine-2 rotates in the anti-clockwise direction. Replacing n_{s2} by $-n_{s2}$ in the above equations and simplifying one can show that the new rotor speed becomes:

$$n_r = \frac{2f}{p_1 - p_2}$$

Thus we can run the set at four possible speeds as follows:

- If the second machine remains idle, by disconnecting its stator supply and shorting the slip-terminating terminals of the first machine, the rotor speed will be $n_r \approx 2f/p_1$.
- If the first machine remains idle, by disconnecting its stator supply and the second machine is supplied from the supply mains, the rotor speed will be $n_r \approx 2f/p_2$.
- If the machines are cascaded as discussed above with both the stator fields moving in the same direction $n_r \approx \frac{2f}{p_1 + p_2}$.
- If the machines are cascaded such that the stator field of the second machine moves in the opposite of the direction as that of the stator field of the first machine then, $n_r \approx \frac{2f}{p_1 - p_2}$.

4.2 speed control by varying the number of poles of the stator

Since induction motor runs at a speed close to the synchronous speed, it is therefore expected if the synchronous speed n_s could be changed then speed of the motor could be varied. We know synchronous speed is inversely proportional to the number of pole, more the number of poles less will be the rotor speed. This method is applicable only to cage induction motor. As cage induction motors can automatically adjust to different number of poles dictated by stator poles. Now the question is how to change the number of poles produced by stator winding? We have learned in module-7 that lay out of a 3-phase winding is carried out for a given number of poles. For example, we know how to make a 3-phase, 4 pole winding. One solution is to put another separate 3-phase

winding which will produce 2 pole in the slots which is already housing the 4-pole winding. If you want have higher speed, energize the 2-pole winding only and if you want to have lower speed energize the 4-pole winding. But this method is not attractive as the utilization of copper is less, size and cost increases. Is it possible to have different number of poles using the same 3-phase winding? The answer is yes and the method is known as consequent pole technique as explained below.

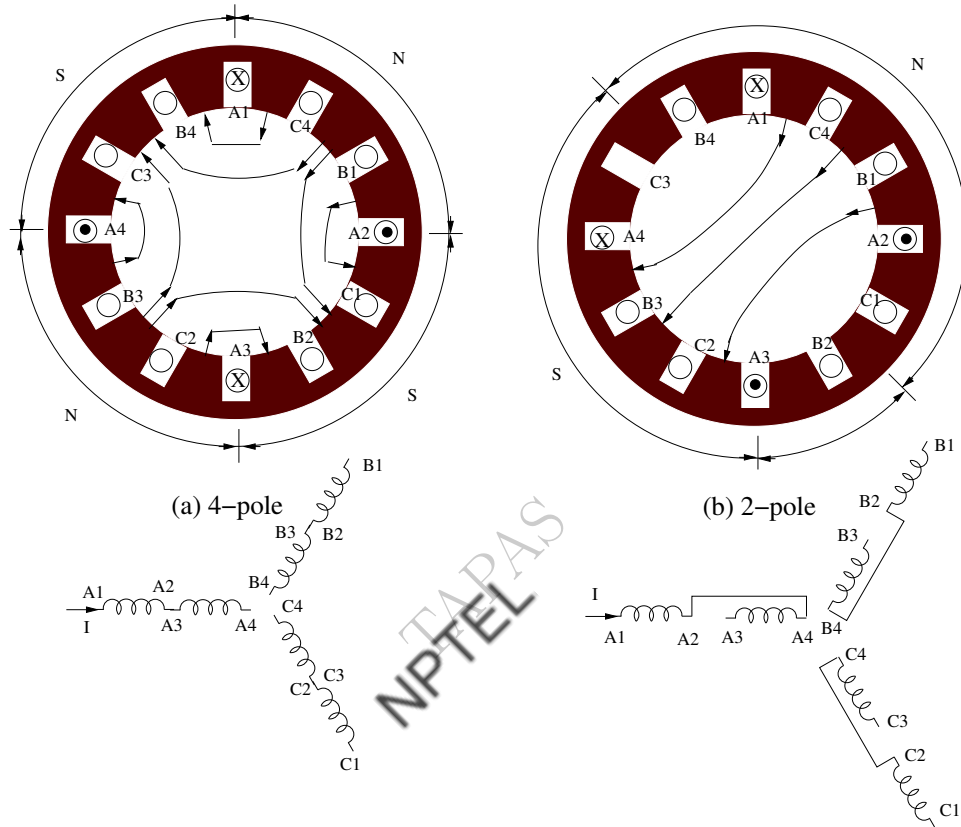


Figure 7: Speed control by consequent pole-technique

Consider a balanced 3-phase, 4 pole, single layer winding as shown in Figure 7(a). as expected, A-phase has two coils with terminals A_1A_2 and A_3A_4 . these two coils are connected in series. To examine how many poles a winding produces, it is sufficient to energize a particular phase with constant current. If current I is passed through A-phase only, it enters through terminal A_1 and leaves A_2 then enters through terminal A_3 and leaves A_4 . Current entering to a conductor is represented as $\otimes$ and leaving a conductor is represented as $\odot$. Looking at the slot current pattern in 7(a) and sketching the lines of forces we can conclude that 4-pole have been produced the winding.

However, if the series connection of these two coils are changed such that current enters through A_2 and is connected to A_4 as shown in Figure 7(b) the current pattern changes because current now enters through A_1 for the first coil and it enters through A_4 for the second coil. Looking at the pattern of lines of force it can be concluded that 2-pole has now be produced. Thus by merely changing the interconnections between the groups of coil of the phases different number poles can be produced. To make this change of connections all the terminals of different groups belonging to all phases must be brought out. Number of poles can be changed in the ratio of 2. For example a 3-phase, 8-pole winding can be transformed to a 3-phase, 4-pole winding by connecting the groups of coil in appropriate manner.

4.3 speed control by varying the supply voltage

In this method, supply voltage is reduced from the rated value V_1 to V_2 , V_3 etc. point of intersection of the torque-speed characteristic of the motor and T_L characteristic of the load determines the speed of operation of the motor. When load torque demand is constant (independent of speed), the range of variation of speed is unto n_3 (corresponding to the reduced voltage V_3) from the initial speed (which is close syn speed) at rated voltage V_1 as shown in Figure 8(a). Although variation of voltage gives smooth control of the speed, the range of it is rather limited and also note T_{max} drastically reduces. Thus large value constant load torque can be supplied. However, if the load is of fan type (i.e., T_L requirement is less at lower speed), range of speed control may be more as depicted in Figure 8(b).

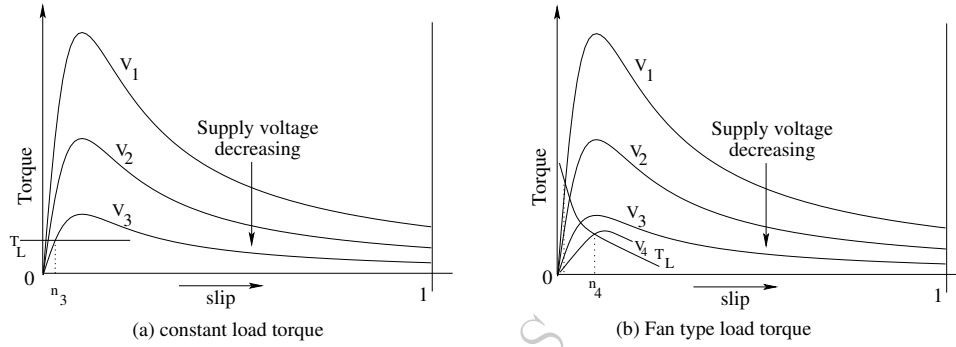


Figure 8: Speed control by controlling supply voltage

4.4 speed control by varying rotor resistance

In this method, external variable resistance is added in the rotor circuit of slip ring induction motor. By varying the value of the resistance smooth speed control of the motor is possible over a wide range. Speed-torque characteristics of the motor and the load are shown in Figures 9(a) and (b). (a) corresponds to constant load torque and (b) corresponds to fan type load. It is observed that the motor can be driven even at very low speed by increasing the rotor resistance. Unlike voltage control, load torque corresponding to full load torque can be supplied over a wide speed range. However, it should be noted that this method is highly inefficient due to the power dissipation in the external resistance.

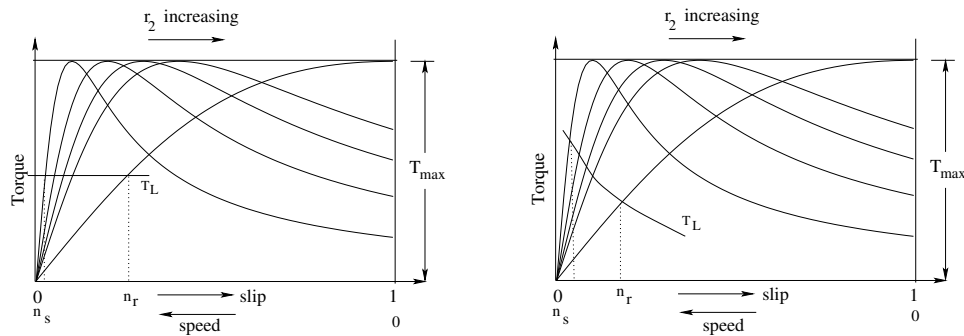


Figure 9: Speed control by varying rotor resistance

4.5 speed control by varying the supply frequency

4.5.1 Varying frequency below rated frequency

It is an elegant method of controlling speed of induction motor smoothly over a wide range. Induction motor is supplied from a variable voltage - variable frequency source with the constrain that v/f ratio remain constant. This is necessary to avoid saturation at frequencies lower than the nominal frequency of the machine. By varying frequency one in fact changes the synchronous speed ($n_s = 2f/p$) of the motor, hence the rotor speed since we know rotor speed will be close to synchronous speed as shown in Figures 10(a) and (b) corresponding to constant load torque and fan type load respectively. As can be seen that smooth speed control is possible over a wide range for full load torques. It should be noted that at very low frequency, stator resistance drop will cause T_{max} lower. One can think of elaborate motor-generator set to get variable voltage - variable

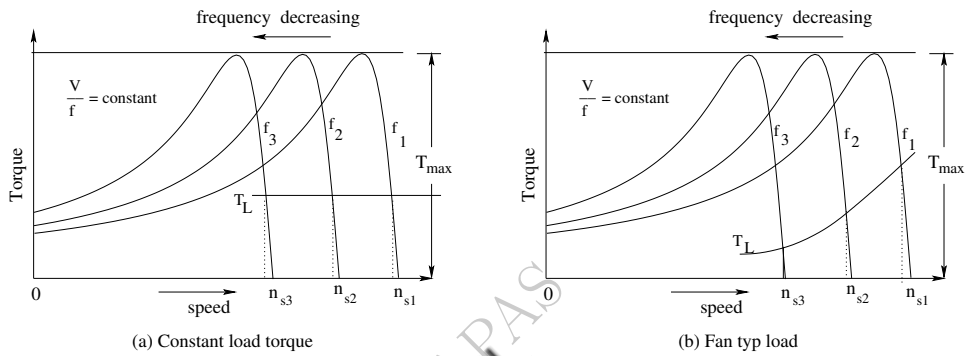


Figure 10: Speed control by varying supply frequency

frequency source to supply induction motor. But such a scheme will require extra machines and space which can not be afforded. In present day, with so much advancement in power electronics converters and inverters, V/f control is possible with a simple controller block. This is explained in the form of schematic block diagram in figure 11 Three phase, fixed voltage, 50 Hz supply is first

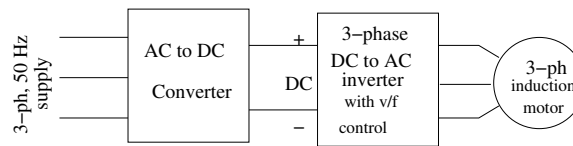


Figure 11: Scheme for variable voltage-variable frequency supply

rectified with the help of an AC to DC converter. The DC obtained is fed to a DC to AC 3-phase inverter. One can control the output voltage and frequency of the inverter such that v/f ratio will remain constant. No extra arrangement of starting of the motor is necessary as one can supply the motor with low voltage at low frequency during starting.

4.5.2 Varying frequency above rated frequency

In this case supply frequency is varied above the rated frequency f_1 . However, as frequency increases one should not apply proportionately increased voltage arguing that flux should be kept constant. This is because we can not apply voltage which is greater than the rated voltage. Thus situation is that applied voltage is kept fixed at rated value and frequency is increased - the consequence is that flux decreases. What happens to the maximum value of the torque? Recall $T_{max} \propto (V/f)^2$, so T_{max} is inversely proportional to the square of the frequency, f^2 . This is shown in figure 12

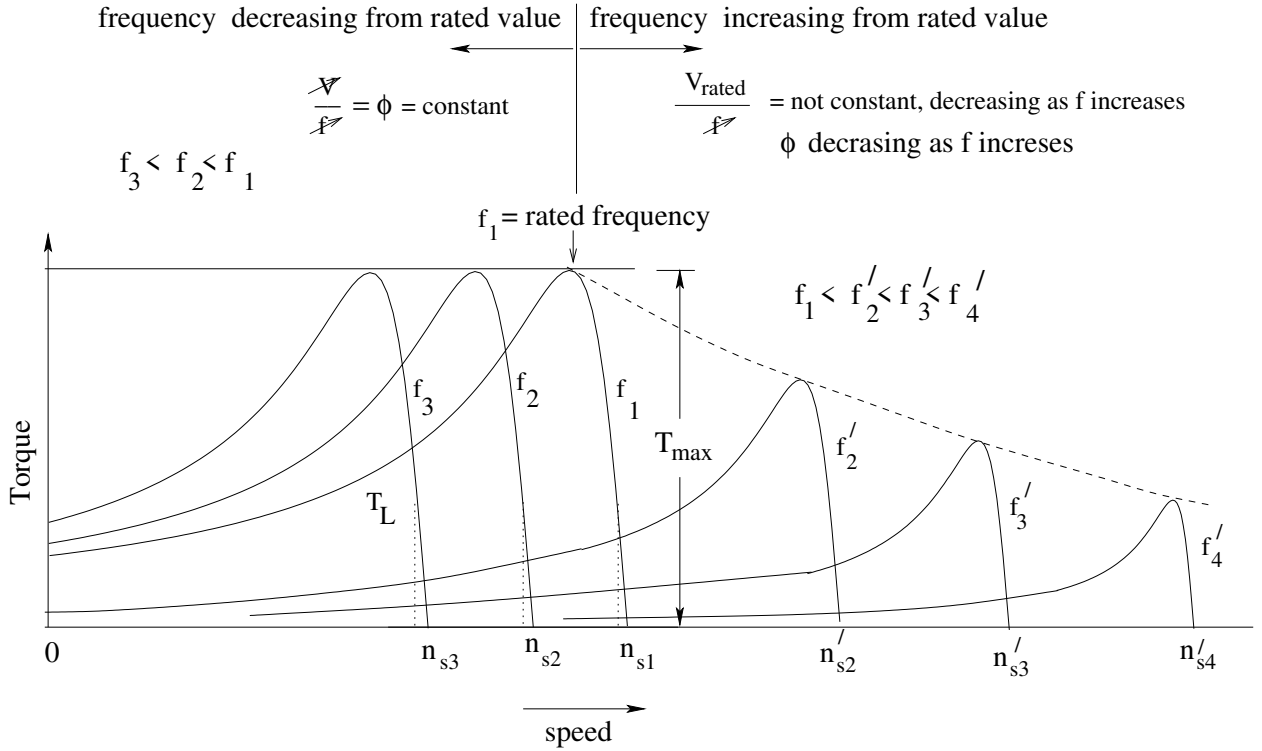


Figure 12: Complete torque-slip characteristics for frequency variation

5 Electrical braking

Sometimes it is necessary to bring a rotating electrical machine quickly to stop. A rotating machine has kinetic energy stored in it. Faster the stored energy is evacuated from the machine and dissipated, quicker the machine will come to stop. For example a machine if simply disconnected from the supply will also eventually come to a stop dissipating its stored kinetic energy as heat in the bearing (frictional torque). Since power dissipated as frictional loss being small, machine will come to stop after some time decided by its moment of inertia. In such a situation a large machine may take longer time to come to a stop. One method is to apply mechanical brake, where stored energy will be primarily dissipated in the brake shoes as heat. However, regular maintenance of the brake shoes are necessary.

To execute electrical braking, electromagnetic torque is to be developed in a direction opposite to the direction of rotation of the motor. Following are the methods by which electrical braking can be accomplished.

1. **Plugging** : In this method any two of the supply terminals are reversed as shown in Figure 13. Suppose the triple pole double throw switch S is connected to the left supply terminals A,B and C and the machine is running as a motor steadily at speed n_r or slip $s = (n_s - n_r)/n_s$. It may be noted that Both the stator field and the rotor moves in the clock-wise direction. If suddenly the switch is quickly thrown to the right, supply phase sequence to the motor gets reversed. Hence the stator field now rotates in the anti-clockwise direction; although the motor continues to run in the clock-wise direction due to inertia. The value of the new slip immediately after execution of plugging is given by:

$$s_b = (-n_s - n_r)/-n_s = (n_s + (1 - s)n_s)/n_s = 2 - s$$

Braking torque can be calculated by replacing s with $2 - s$ in the torque expression. When

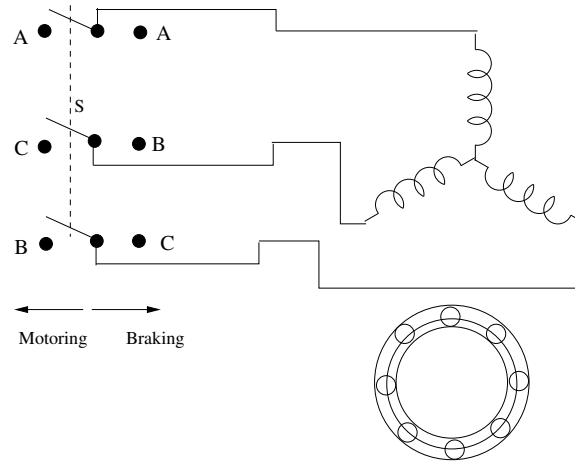


Figure 13: Braking torque under plugging

plugging is executed, induced voltage & current in the rotor will be high as slip is now $(2 - s)$. Also reflected current in the stator coils will be high. For slip ring induction motor, additional rotor resistance is connected which not only limit the current during plugging but also increases braking torque. The situation is explained in Figure 14(a). Suppose the machine is operating as motor at point A with slip $= s$. The dotted torque-slip characteristic is drawn with additional external resistance connected in the rotor circuit. As soon as plugging is executed, slip becomes $(2 - s)$ and operating point shifts to B from A. Torque (i.e., braking torque T_b) developed by the machine should now be read from the dotted curve. From physical

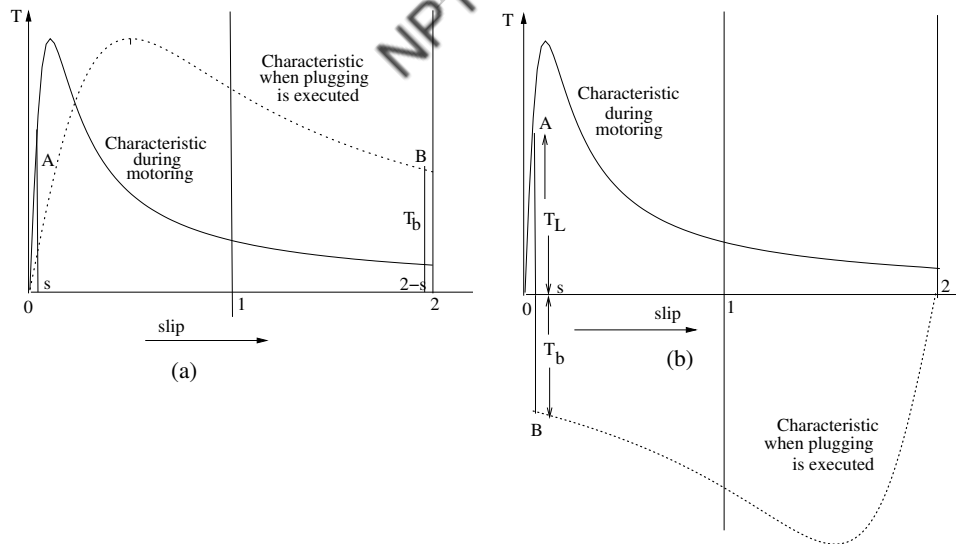


Figure 14: Braking torque under plugging

consideration we already know that this torque will act in the opposite direction of rotation. In Figure 14(b) The dotted curve of figure 14(a), is now flipped about $s = 1$ vertical line and then inverted (mirror image) about the horizontal slip axis and we get Figure 14(b). The advantage of this exercise is that we can read the value of T_b much more easily. For example for initial motoring slip s (point A), drop a perpendicular from A on to the slip axis and extend it so that it intersect at B on dotted T-s characteristic. So T_b can be read as indicated. It may be noted that supply should be disconnected from the machine when speed approaches zero; otherwise it will start rotating in the opposite direction.

2. D.C. dynamic braking: Let the machine be running as a motor when the switch S is connected

to 3-phase supply (refer to Figure 15). If the switch S is suddenly moved to the right, then stator terminals get disconnected from the a.c supply and a direct current flows through the stator coils. The stator field produced by this constant stator current will be stationary in space with respect to the stator. Since rotor is already rotating and the rotor circuit is closed, rotor will act as generator dissipating power in the rotor resistance. The electromagnetic torque however will be acting in the opposite direction of rotor rotation. The magnitude of the braking torque decreases as the speed decreases.

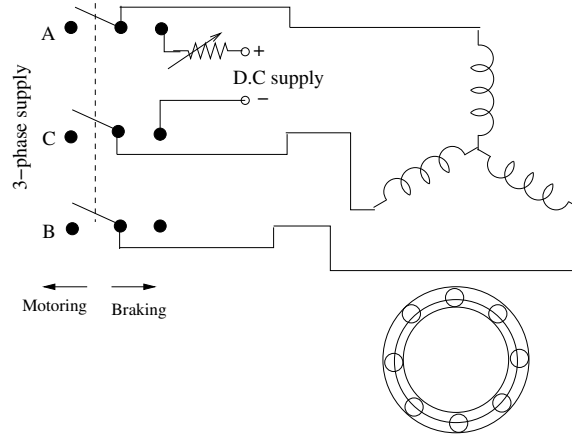


Figure 15: DC dynamic braking

The estimation of the braking torque during d.c dynamic braking can be carried out as follows. Suppose the machine was running as a motor with slip s and the rotor speed was $n_r = (1-s)n_s$ in the clockwise direction, where n_s is the synchronous speed decided by the mains frequency f . Immediately after stator supply is switched over to D.C, stator field F_s becomes stationary in space and rotor continues to run at n_r as depicted in Figure 16(a). To calculate the

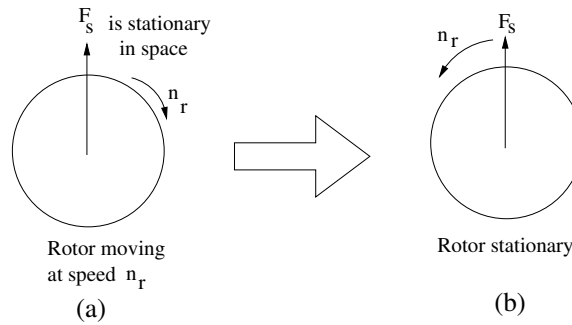


Figure 16: Scenario when switched over to DC

electromagnetic torque developed now, we can imagine that F_s is moving in the counter-clockwise direction with speed n_r and the rotor is stationary as shown in 16(b). The problem is therefore to calculate the starting torque of an equivalent 3-phase induction motor (shown in (b)), the synchronous speed of which is $n_r = (1-s)n_s$. Therefore the supply frequency of this equivalent machine should be

$$f_{eq} = pn_r/2 = pn_s(1-s)/2 = (1-s)f$$

Therefore all the reactances in the equivalent circuit of the equivalent machine should be multiplied by $(1-s)$ as shown in Figure 17. Now we should find out the value of the equivalent AC current I_{ac} which will produce same mmf as by the direct current I_{dc} . Let us calculate the

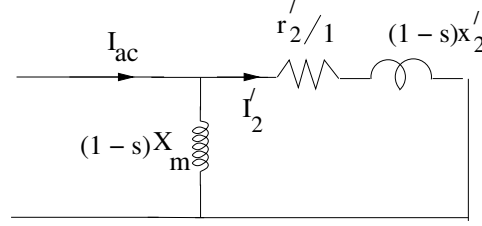


Figure 17: Equivalent circuit of the equivalent machine

resultant mmf When DC supply is connected in such a way that I_{dc} flows through two phase A and C with B-phase current being zero. Assuming that if current enters through terminal 1 of a phase (in this case A1) mmf vector is drawn outwardly along the axis of the coil. The two mmf vectors due to A and C phase are drawn. The strength of each mmf will be NI_{dc} . and their resultant is $\sqrt{3}NI_{dc}$. In case of 3-phase winding carrying balanced AC current, the strength of the resultant field is already known to be $(3/2)NI_{max}$ where I_{max} is the maximum value of the sinusoidal phase current. In terms of rms value of the current this comes out to be $(3/\sqrt{2})NI_{rms}$

In this case we wish to get the value equivalent rms AC current I_{ac} . Thus equating the resultant field strengths of DC and AC, we get:

$$\frac{3}{\sqrt{2}}NI_{ac} = \sqrt{3}NI_{dc}$$

$$\text{or, } I_{ac} = \sqrt{\frac{2}{3}}I_{dc} \quad (14)$$

Thus knowing I_{dc} and we can estimate equivalent I_{ac} . Once we get I_{ac} , we can calculate I'_2 from the equivalent circuit shown in Figure 17 as follows.

$$I'_2 = \frac{(1-s)X_m}{\sqrt{(r'_2)^2 + (1-s)^2(X_m + x'_2)^2}} I_{ac}$$

Therefore the braking torque T_b (which is same as the starting torque of the equivalent machine supplied from a 3-phase balanced source at frequency $f_e = pn_r/2$) is given by:

$$T_b = 3(I'^2_2 r'_2)/2\pi n_r$$

putting the value I'_2 , $T_b = \frac{1}{2\pi n_r} \frac{(1-s)^2 X_m^2 r'_2}{r'^2_2 + (1-s)^2 (X_m + x'_2)^2} I_{ac}^2$

but we know, $n_r = (1-s)n_s$ where, s is the slip of the actual motor when dynamic braking was executed

$$\text{Thus, braking torque is, } T_b = \frac{1}{2\pi n_s} \frac{(1-s)^2 X_m^2 r'_2}{r'^2_2 + (1-s)^2 (X_m + x'_2)^2} I_{ac}^2 \quad (15)$$

3. Regenerative braking: This kind of electromagnetic braking torque is imposed on the shaft of the machine whenever its speed becomes super-synchronous (i.e., more than the synchronous speed). Note that the machine remains connected to the supply all along. We have seen that as a motor the rotor speed is sub-synchronous. Imagine an induction motor is driving vehicle (say a train). On flat track it is indeed running as a motor drawing power from the 3-phase supply. If the train now moves a down hill, its speed may go up beyond synchronous speed

due to component of gravitational force. But the moment speed becomes super synchronous, slip of the machine becomes negative. This indicates that the machine must be acting as a generator feeding power back to the supply. Hence the name regenerative braking.

Thus we see that an induction machine operates as a motor in the slip range of $0 < s \leq 1$. During plugging slip will be greater than 1. Induction machine connected to bus may also operate as a generator with negative value of slip. Figure 18 shows the torque slip characteristic of the machine over a slip range of $-2 < s < 2$ at rated voltage.

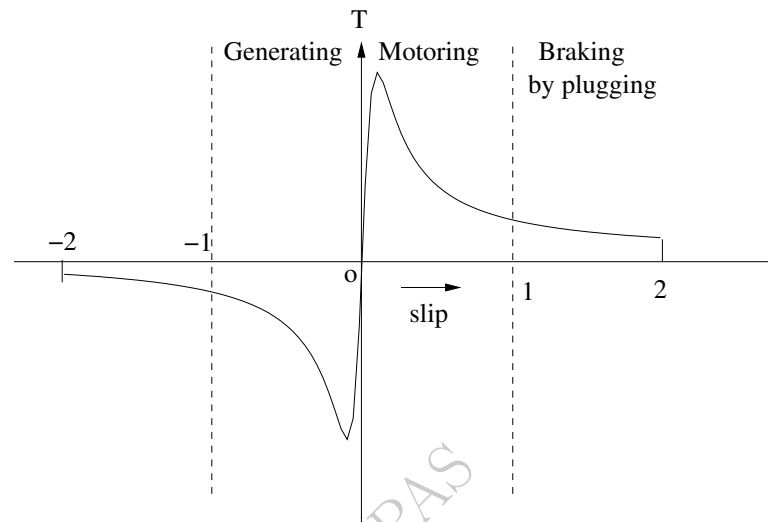


Figure 18: Torque slip characteristic for $-2 < s < 2$

6 Problems

1. A 3-phase, 4 pole, 50 Hz induction motor is running steadily at 1455 rpm. If now any two supply terminals are reversed what is the slip now?

Solution:

$$\text{Syn speed when running as motor, } n_s = 1500$$

$$\text{Therefore slip as a motor, } s = (1500 - 1455)/1500 = 0.03$$

$$\text{Syn speed with two supply terminals reversed, } n_s = -1500$$

But rotor speed can not change instantaneously

$$\therefore \text{ new slip is } = (-1500 - 1455)/(-1500) = 1.97$$

2. Two 3-phase, 50 Hz mechanically coupled induction motors are connected in cascade. Number of poles of the machines are respectively 6 and 8. Calculate various speeds at which the set can be run?

Solution:

$$\text{When first machine alone is energized, speed } \approx 120 \times 50/6 = 1000 \text{ rpm}$$

$$\text{When second machine alone is energized, speed } \approx 120 \times 50/8 = 750 \text{ rpm}$$

$$\text{Cascaded \& fields of motors move in same direction, speed } \approx 120 \times 50/(8 + 6) = 428.5 \text{ rpm}$$

$$\text{Cascaded \& fields of motors move in opp. direction, speed } \approx 120 \times 50/(8 - 6) = 3000 \text{ rpm}$$

3. Suppose a 3-phase, 440 V, 4-pole, 50 Hz, induction motor is supplied from a variable voltage, variable frequency source. The motor supplies a constant load torque. Initially it is operating at rated voltage and frequency. Answer the following:
- (i) What is the approximate initial speed?
 - (ii) To run the machine approximately at half the speed, supply frequency is halved. Should the magnitude of the voltage be also halved? If yes, why?
 - (iii) With both voltage and frequency halved, what is the approximate final speed?
 - (iv) During the course of settling to the final speed from the initial speed (obtained in (i)), explain the mode in which the machine was operating.

Solution:

- (i) No slip value is given. Speed will be close to 1500 rpm but slightly less than it.
- (ii) To run it close to 750 rpm, supply frequency is to be made 25 Hz and supply voltage also to be halved. Otherwise saturation will take place and magnetizing current will be prohibitively large. In fact V/f ratio must be kept constant in such scheme.
- (iii) Final speed will be close to 750 rpm but slightly less than it.
- (iv) When the motor was supplied from 50 Hz, speed was close to 1500 rpm. Imagine that frequency is now made 25 Hz. While synchronous speed becomes 750 rpm, rotor speed can not change instantaneously. So till it reaches the final speed, machine speed is super-synchronous and it will operate in the regenerative braking mode. The difference of kinetic energy between the two steady speeds will be fed back to the supply.